Approximate Equivalence of Transduction

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Motivations

1. Data Exchange
2. xslt Transformations
3. Model versus Specification

Outline: Preliminaries

1. Word Edit Distance with Moves
2. Statistics
   - Uniform Statistic of a Word
   - Link Statistics-Distance
   - Uniform Statistic of a Loop
3. Word Transducer
   - Definition
   - Relation defined by a Transducer
   - Powering
   - Graph of the strongly connected components

Elementary operations

- Insert a letter : $aaabbb \xrightarrow{ins} aaa\mathbf{c}bbb$
- Remove a letter : $aaacbb\mathbf{b} \xrightarrow{del} aaacbb$
- Change a label : $aaacb\mathbf{b} \xrightarrow{swi} aaacbc$
- Move a subword : $a\mathbf{aa}cbc \xrightarrow{mov} acb\mathbf{aa}c$

Edit Distance with moves

1. $\mathsf{dist}(w_1, w_2) = \min\{p \mid w_1 \xrightarrow{o_1} \ldots \xrightarrow{o_p} w_2\}$
2. $\mathsf{d}(w_1, w_2) = \dfrac{\mathsf{dist}(w_1, w_2)}{\max(|w_1|, |w_2|)}$
3. $\mathsf{dist}(w_1, L) = \min_{w_2 \in L} \mathsf{dist}(w_1, w_2)$
4. $\mathsf{dist}(L_1, L_2) = \min_{w_1 \in L_2, w_2 \in L_2} \mathsf{dist}(w_1, w_2)$

Edit Distance with moves

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4. $\mathsf{dist}(L_1, L_2) = \min_{w_1 \in L_2, w_2 \in L_2} \mathsf{dist}(w_1, w_2)$

Examples

1. $\mathsf{dist}(aaabbb, bbbaac) = 2$
2. $\mathsf{d}(aaabbb, bbbaac) = 1/3$
3. $\mathsf{dist}(aaabbb, a^*) = 3$
4. $\mathsf{dist}\left((aaa)^+.(bbbbbbbb)^+ \ , \ (bbbbbbbb)^*.(aaa)^+\right) = 1$

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Let Σ be a binary alphabet, $w \in \Sigma^*$ of length n and $k \in \mathbb{N}^+$

Statistic of a Word : ustat_k

$$\mathsf{ustat}_k(w) = \frac{1}{n-k+1}. \begin{pmatrix} \#n_1 \\ \#n_2 \\ \vdots \\ \#n_{2^k} \end{pmatrix} \text{ where } \begin{array}{c} \#n_1 : \text{number of } 00\ldots0 \\ \#n_2 : \text{number of } 0\ldots01 \\ \vdots \\ \#n_{2^k} : \text{number of } 1\ldots11 \end{array}$$

Example

$w = 001010101110$, length $n = 12$, $k = 2$, $n - k + 1 = 11$ blocks

$$\mathsf{ustat}(w) = \frac{1}{11} \begin{pmatrix} 1 \\ 4 \\ 4 \\ 2 \end{pmatrix}$$

Let Σ be a alphabet, $w \in \Sigma^*$ of length n and $k \in \mathbb{N}^+$

Statistic of a Word : ustat_k

$$\mathsf{ustat}_k(w) = \frac{1}{n-k+1} \cdot \begin{pmatrix} \#n_1 \\ \#n_2 \\ \vdots \\ \#n_{|\Sigma|^k} \end{pmatrix}$$

Main Properties

1. $\mathrm{d}(w, w') \approx |\mathsf{ustat}(w) - \mathsf{ustat}(w')|_1$ for words of similar length
2. $|\mathsf{ustat}(w) - Y(w)|_1 \leq \varepsilon$ for $Y(W)$ the ustat vector on N samples
3. distance between words
   - NP-complete
   - Testable, $\mathcal{O}(1)$:
     Sample N subwords of length k: $Y(w)$ and $Y(w')$.
     If $|Y(w) - Y(w')|_1 < \varepsilon = \frac{1}{k}$ accept, else reject

Limit Statistic of a loop

$$\text{Let } L = w^* \ , \ |w| = q$$

$$\mathcal{H}_{\text{lim}}(L)[u] = \Pr_{j=1,\dots,q}\left[w[j].w[j{+}1 \text{ Mod } k]\dots w[j{+}k{-}1 \text{ Mod } k] = u\right]$$

Example

$$L = (0001)^*, k = 2, \mathcal{H}_{\text{lim}}(L) = \begin{pmatrix} 1/2 \\ 1/4 \\ 1/4 \\ 0 \end{pmatrix}$$

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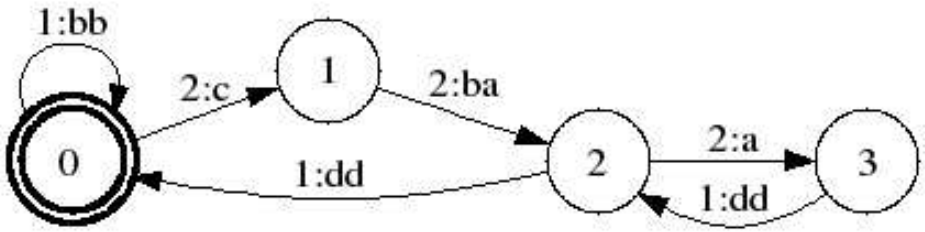
Word Transducers

A word transducter $\mathscr{T}$ is a tuple $(\Sigma, \Sigma', Q, I, F, \delta)$ where

- Σ in the input alphabet, Σ' the output alphabet
- Q a set of states, $I \in Q$ an initial state, $F \subseteq Q$ final states
- $\delta : Q \times (\Sigma \cup \Lambda) \times (\Sigma')^* \times Q$ the transition fonction

Example of a Word Transducer

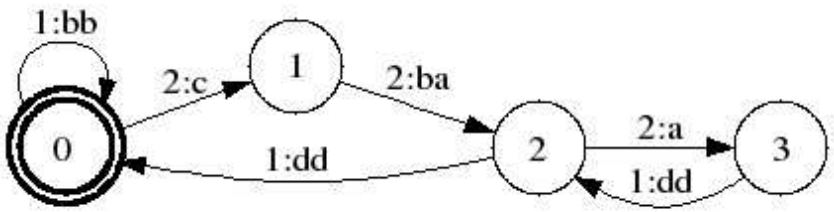
- $\Sigma = \{1, 2\}, \Sigma' = \{a, b, c, d\}$
- $Q = \{0, 1, 2, 3\}, I = \{0\}, F = \{0\}$
- $\delta = \{(0, 1, bb, 0), (0, 2, c, 1), (1, 2, ba, 2), (2, 1, dd, 0), (2, 2, a, 3), (3, 1, dd, 2)\}$



Relation of a Word Transducer $\mathscr{T}$

For two words w_1 and w_2, (w_1, w_2) is in the relation defined by $\mathscr{T}$, if there is a run from the initial state to a final state inputing w_1 and outputing w_2. [1]

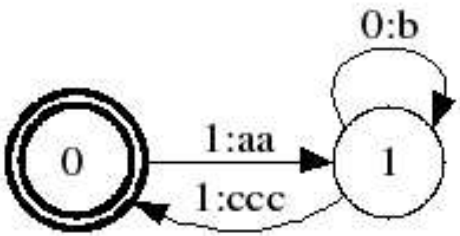
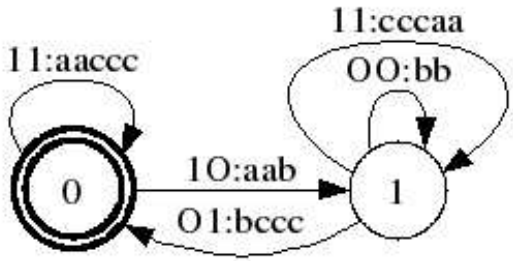
Example



$(\Lambda, \Lambda), (1, bb), (22211, cbaadddd)$ are in the relation defined by $\mathscr{T}$

[1] We then note $(w_1, w_2) \in \mathscr{T}$

Example : Squaring

a transducer	its square
	

k-Powering of a Transducer

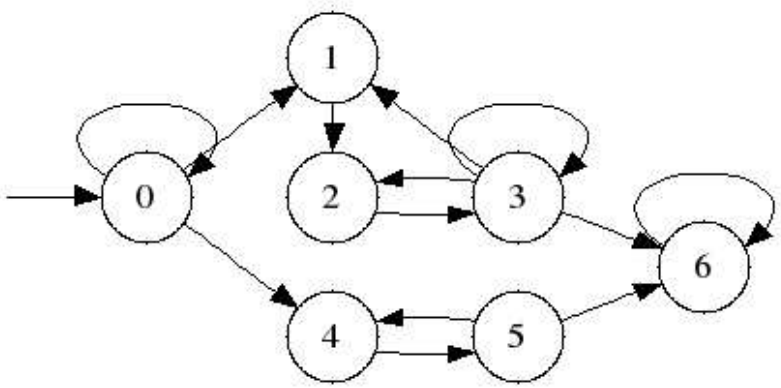
Let $\mathscr{T} = (\Sigma, \Sigma', Q, I, F, \delta)$ a transducer and $k \in \mathbb{N}$.

$$\mathscr{T}^k = (\Sigma^k, \Sigma', Q, I, F, \delta^k) \text{ where}$$

$(q_1\ ,\ a_1a_2...a_k\ ,\ w\ ,\ q_{k+1}) \in \delta^k$ iff there exist $q_1, ..., q_k \in Q$ and $w_1, ..., w_k \in (\Sigma')^*$ such that $w = w_1.w_2...w_k$ and for all i in $\{1, ..., k\}$, $(q_i, a_i, w_i, q_{i+1}) \in \delta$.

For G the graph of a (delabeled) automaton or transducer, $\widehat{G}$ denotes the graph of strongly connected components of G

Example



G

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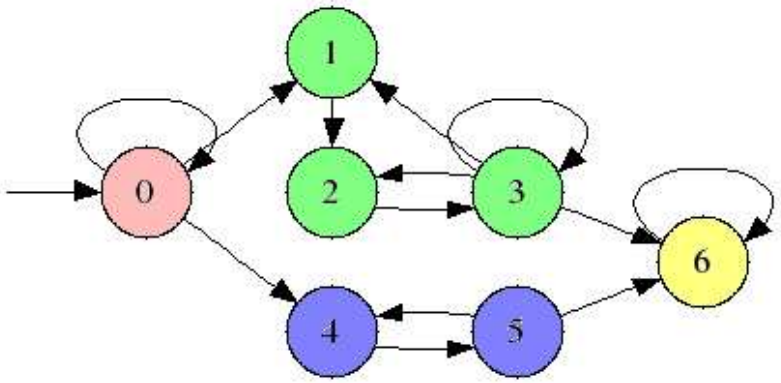
G



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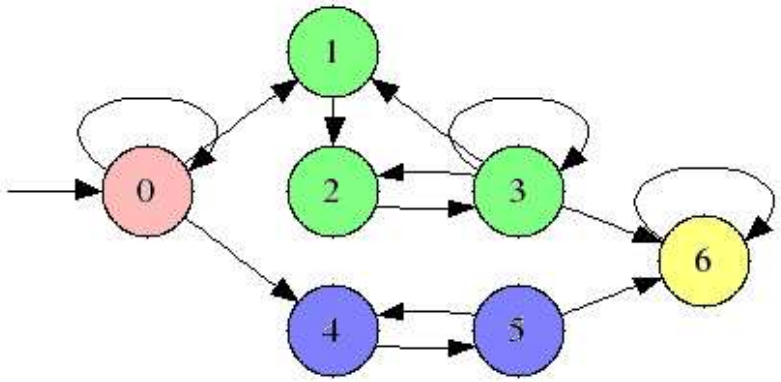
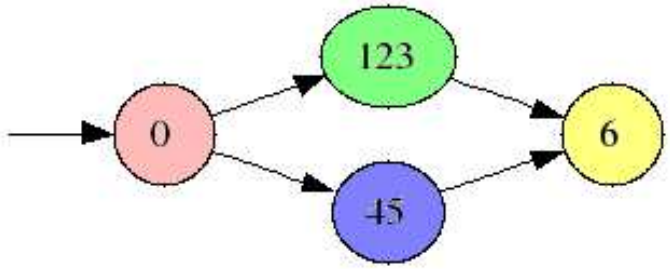


$\widehat{G}$



Outline: Equivalences

4 warm up

5 definition

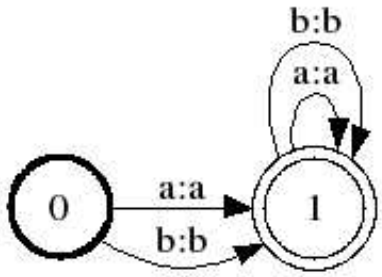
6 exact case

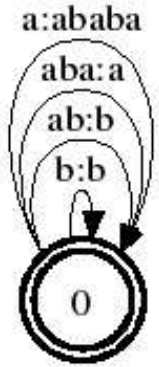
7 ε-equivalence
- two transducers ε-close
- statistical representation
- In the relation => close to the embedding
- In the embedding => close to the relation
- counterexamples : sources of non ε-equivalence

The intersection is undecidable

Reduction to the Post Correspondance Problem

- an instance: $\left(\begin{array}{c} a \\ ababa \end{array}\right), \left(\begin{array}{c} aba \\ a \end{array}\right), \left(\begin{array}{c} ab \\ b \end{array}\right), \left(\begin{array}{c} b \\ b \end{array}\right)$
- a solution: $\left(\begin{array}{c} aba \\ a \end{array}\right)\left(\begin{array}{c} b \\ b \end{array}\right)\left(\begin{array}{c} a \\ ababa \end{array}\right)\left(\begin{array}{c} b \\ b \end{array}\right)\left(\begin{array}{c} aba \\ a \end{array}\right)$





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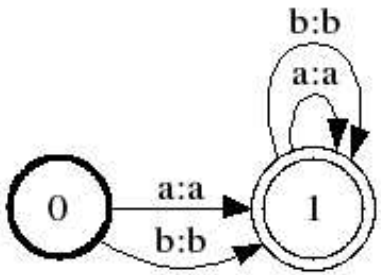

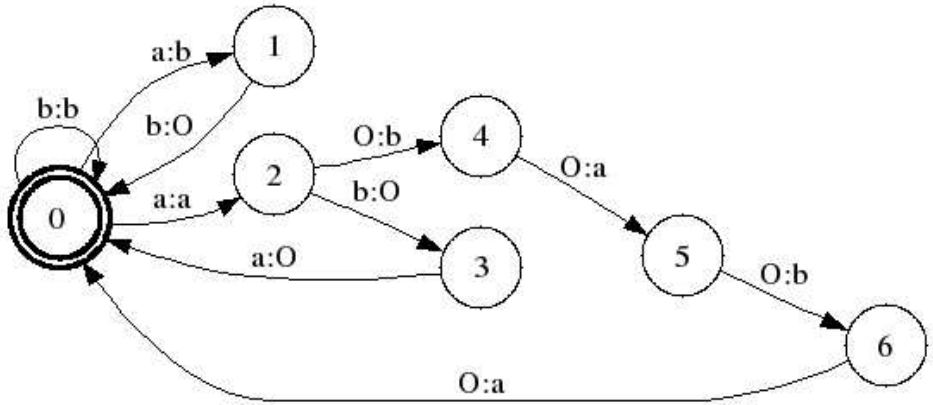

From $\mathscr{T}_1 \subseteq \mathscr{T}_2$ to $\mathscr{T}_1 \subseteq_\varepsilon \mathscr{T}_2$

1. **exact inclusion:** $(w, w') \in \mathscr{T}_1 \implies (w, w') \in \mathscr{T}_2$
2. **ε-inclusion:** $(w, w') \in \mathscr{T}_1 \implies \exists\, (\widetilde{w}, \widetilde{w'}) \in \mathscr{T}_2$ such that, for the classical edit distance with moves
   - $\widetilde{w}$ is ε-close to w *i.e.* $\mathrm{d}(w, \widetilde{w}) \leq \varepsilon$
   - $\widetilde{w'}$ is ε-close to w' *i.e.* $\mathrm{d}(w', \widetilde{w'}) \leq \varepsilon$

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Finer ? Coarser ?

exact inclusion $\implies$ ε-inclusion

Equivalence is undecidable

1. The equivalence problem for non-deterministic transducers (one way, finite state) is undecidable even when they cannot read or write the empty string

   T. V. Griffiths, “The unsolvability of the Equivalence Problem for Lambda-Free nondeterministic generalized machines”,1968.

2. The equivalence problem for 1-free[2] input deterministic[3] sequential transducers is undecidable

   J. Karhumäki and L. P. Lisovik, “On the Equivalence of Finite Substitutions and Transducers”,1999, (Corollary 1).

[2] Transitions do not output the empty string

[3] The (input) underlying automaton is deterministic

Decidability

1. The equivalence for deterministic transducers is decidable
   Bird 1973, Vaillant 1974
2. The equivalence for n-tape deterministic finite automata is decidable Harju & Karhumäki, 1991
3. The equivalence for sequential transducers satisfying the prefix condition[4] is decidable Karhumäki & Lisovik, 1999
4. The equivalence on an NR set of strings for two deterministic two-way finite state transducers is decidable
   Engelfriet, Maneth, 2005

[4] Avoid two transitions from the same state, the same input letter, and an output different and prefix of the other

PSPACE-hardness

Equivalence is at least PSPACE-hard, even restricted to identity edges

proof

as hard as the equivalence of non-deterministic finite automata.

PSPACE-completeness for a weak form of non-determinism

The equivalence of deterministic two-way (sequential) transducers which are allowed to make some finite number of non-deterministic moves is a PSPACE-complete problem

pointed out by Gurari, 1982

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- In the relation => close to the embedding
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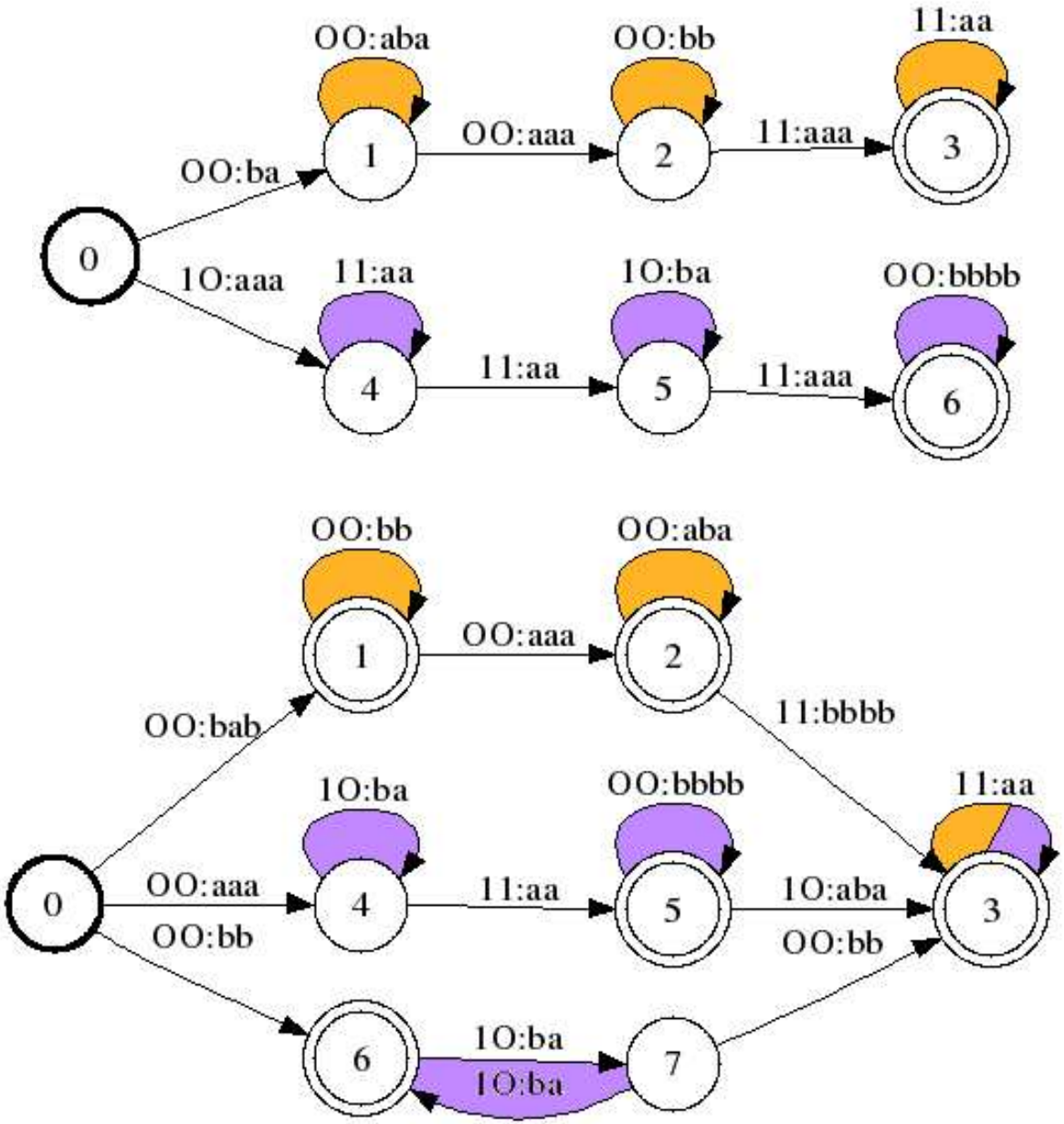
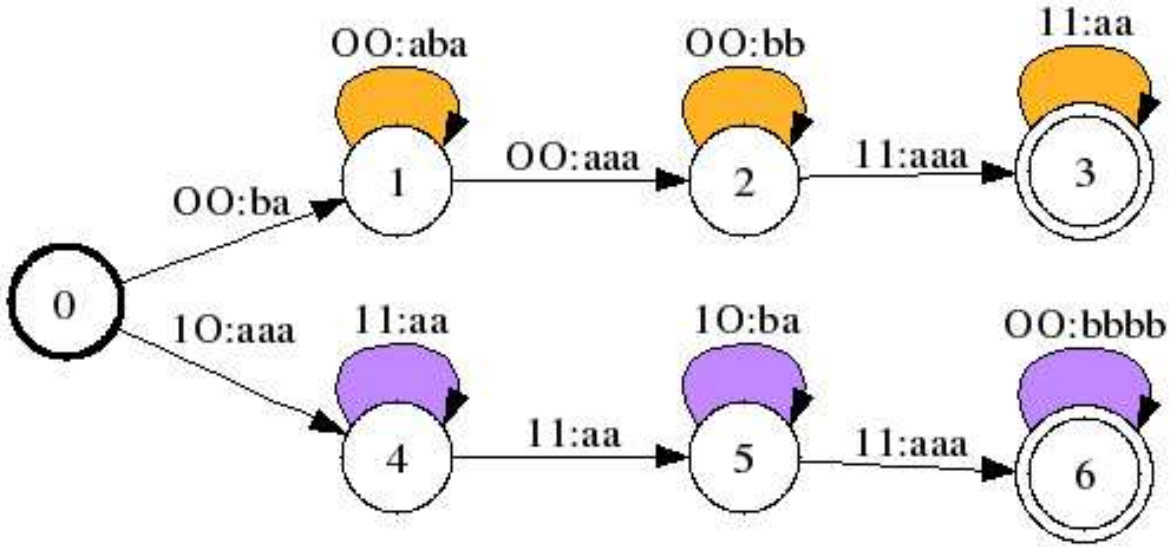

OO:aba
OO:bb
11:aa
OO:ba
OO:aaa
11:aaa
0
1
2
3
1O:aaa
11:aa
1O:ba
OO:bbbb
4
11:aa
5
11:aaa
6
OO:bb
OO:aba
1
OO:aaa
2
OO:bab
11:bbbb
1O:ba
OO:bbbb
11:aa
0
OO:aaa
4
11:aa
5
1O:aba
3
OO:bb
OO:bb
6
1O:ba
7
1O:ba

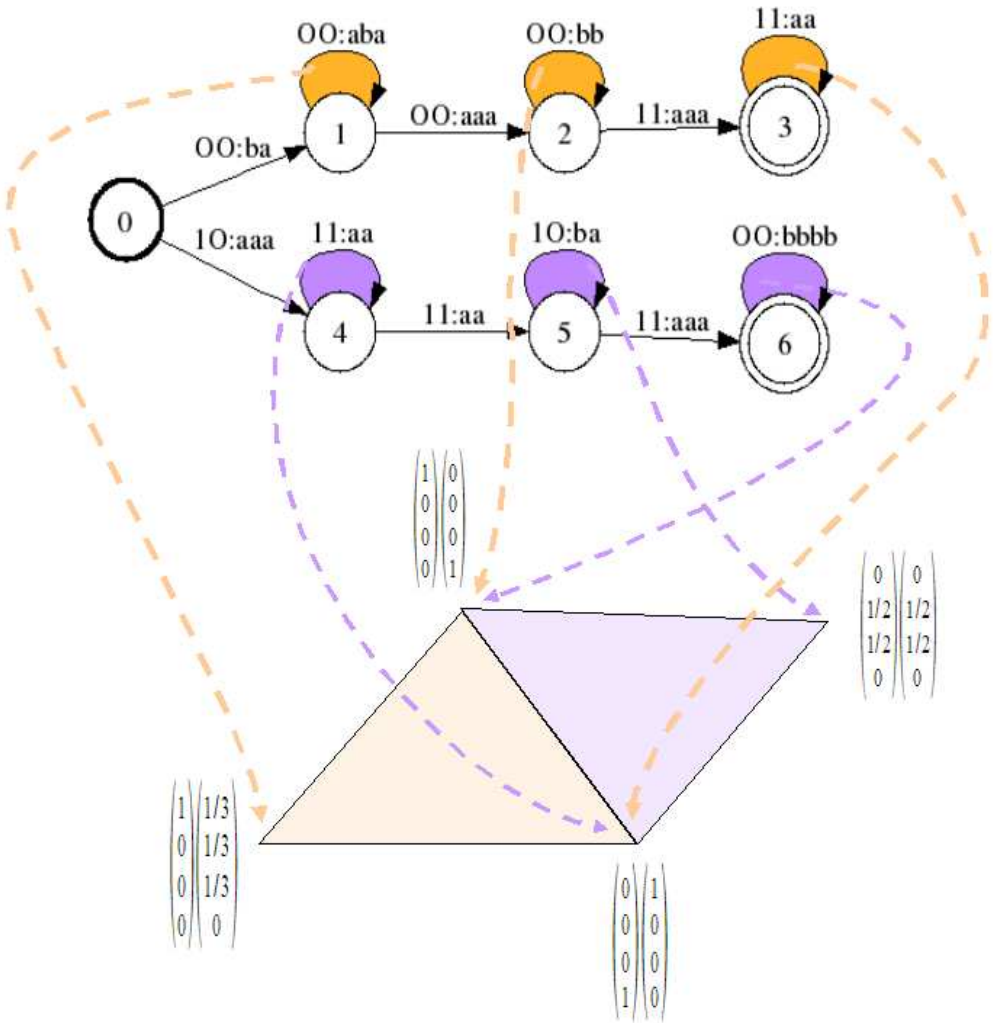
Forget finite contributions



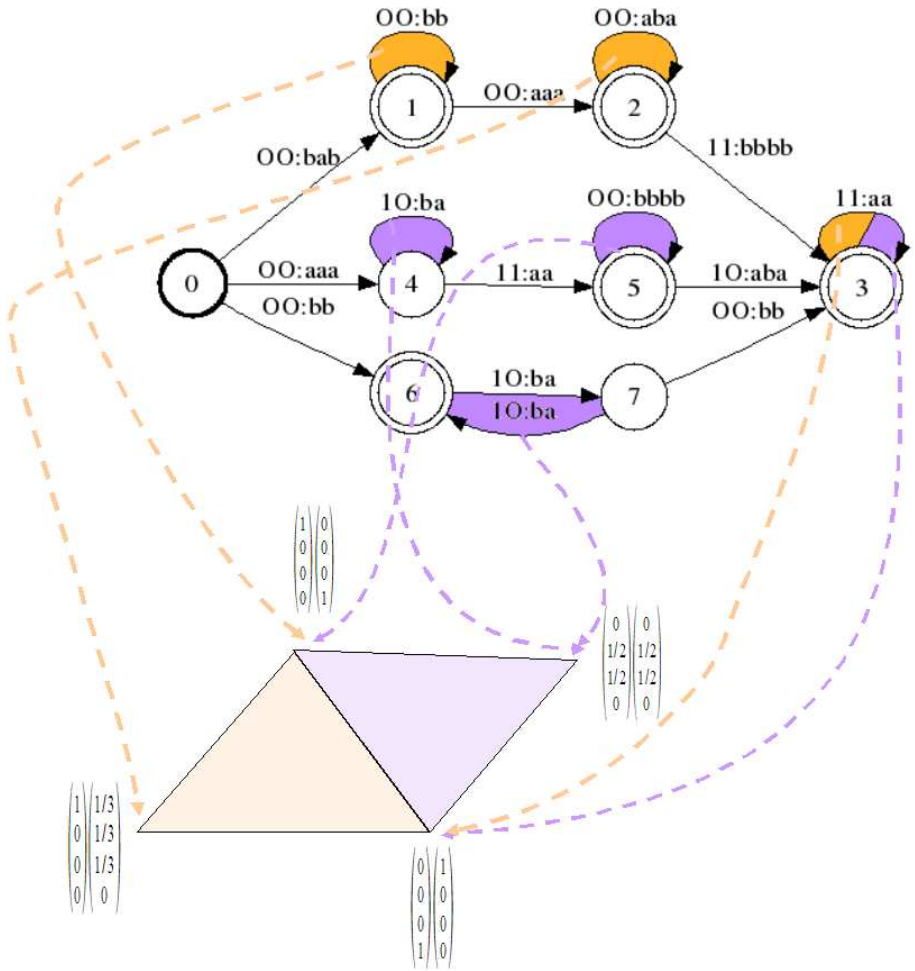
In essence:

$$t = (00 : aba)^*.(00 : bb)^*.(11 : aa)^* + (11 : aa)^*.(10 : ba)^*.(00 : bbbb)^*$$

$\mathscr{T}_1$

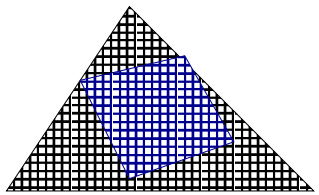

OO:aba
OO:bb
11:aa
OO:aaa
11:aaa
OO:ba
1
2
3
0
1O:aaa
11:aa
1O:ba
OO:bbbb
4
11:aa
5
11:aaa
6

$\mathscr{T}_2$


OO:bb
OO:aba
1
OO:aaa
2
OO:bab
11:bbbb
1O:ba
OO:bbbb
11:aa
0
OO:aaa
4
11:aa
5
1O:aba
3
OO:bb
OO:bb
6
1O:ba
7
1O:ba

ε-Inclusion

- Apply the k-powering of $\mathscr{T}_1$ and $\mathscr{T}_2$, $k = 1/\varepsilon$:
  - Matrix multiplication
- compute the embeddings $\mathcal{H}_\varepsilon = (H_\varepsilon^I, H_\varepsilon^O)$ in the statistical space:
  - analyze paths π in $\widehat{\mathscr{T}_i^k}$
  - convex hulls for π-compatible loops
- $H_\varepsilon^1 \subseteq H_\varepsilon^2$?:
  - H_ε^i is a union of polytopes
  - inclusion of the grids of step ε



- check lengths

$\Longrightarrow$ PSPACE tester for every fixed $\varepsilon > 0$

Assume $(w, w') \in \mathscr{T}^k$

- let π be a path in $\widehat{\mathscr{T}^k}$ s.t. $w' \in \mathscr{T}^k_\pi(w)$
- following π, $w = v_0 u_1 u_2 \dots u_l v_l$,
  $|v_0|, |u_1|, \dots |u_l|, |v_l| \leq m.k$, $\{u_1, \dots, u_l\}$: π-compatible
- $\{u_1, \dots, u_l\} = \{u'_1, \dots, u'_b\}$ with $b \leq a^k$
- take $\widetilde{w} = v_0 (u'_1)^{k_1} v_1 (u'_2)^{k_2} \dots v_{b-1} (u'_b)^{k_b} v_b$ with
  $v_0 v_1 \dots v_b$: witness from compatibility,
  v_i: sequences of less than m edges.
- dist$(w, \widetilde{w}) \leq (b+1)m + \varepsilon n$
- take $\widetilde{w'} = out(v_0)(out(u'_1))^{k_1} out(v_1) \dots (out(u'_b))^{k_b} out(v_b)$
- $(\widetilde{w}, \widetilde{w'}) \in \mathscr{T}^k$
- dist$(w', \widetilde{w'}) \leq maxout(b+1)m + \varepsilon n$

$(\mathsf{ustat}(\widetilde{w}), \mathsf{ustat}(\widetilde{w'}))$ is close to the polytope

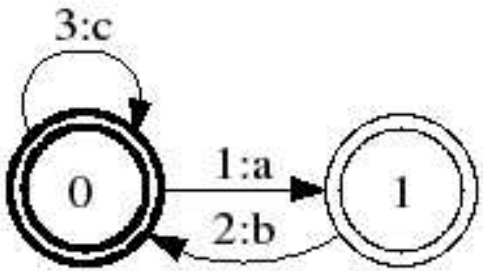
- $\widetilde{x} = \mathsf{ustat}(\widetilde{w})$, $\widetilde{x'} = \mathsf{ustat}(\widetilde{w'})$
- $\beta_i = \frac{k_i}{\sum_i k_i}$
- $\beta = (\beta_1, \ldots, \beta_b)$
- $x = Renorm(\sum_{i=1}^{b} \beta_i l_{in}(B_i) \mathcal{H}_{lim_{in}}(B_i))$
- $x \in \mathcal{H}^I$
- $|x - \widetilde{x}| \leq \frac{(b+1)(3(k-1)+m.p)}{\widetilde{n}}$
- $x' = Renorm(\sum_{i \in b} \frac{l_{out}(B_i)}{l_{in}(B_i)} \beta_i \mathcal{H}_{lim_{out}}(B_i))$
- $(x, x') \in \mathcal{H}$
- $|x' - \widetilde{x'}| \leq \frac{maxout((b+1)(3(k-1)+m.p))}{\widetilde{n'}}$

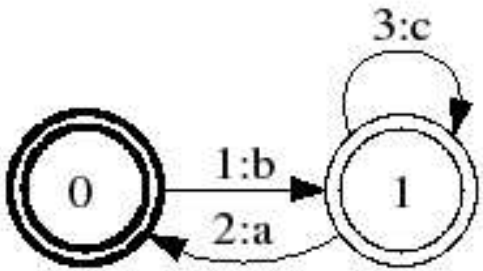
Assume $(\lambda, \lambda') \in \mathcal{H}$

- take sets of summits T, (λ, λ', π)-compatibles:
  - $\lambda = \sum_{i \in T} \lambda_i \mathcal{H}_{lim_{in}}(B_i)$
  - $\lambda' = \sum_{i \in T} \lambda'_i \mathcal{H}_{lim_{out}}(B_i)$
  - $\lambda' = Renorm\left(\frac{l_{out}(B_i)}{l_{in}(B_i)}\lambda_i\right)$
- approximate λ_i by rationals α_i
- denormalize $\widetilde{\lambda_i} = c.\frac{l_{out}(B_i)}{l_{in}(B_i)}\alpha_i$
- take $w = u_0(in(B_1))^{\widetilde{\lambda_1}} u_1 (in(B_2))^{\widetilde{\lambda_2}} u_2 \dots u_{|T|}$
- take $w' = u'_0(out(B_1))^{\widetilde{\lambda_1}} u'_1 (out(B_2))^{\widetilde{\lambda_2}} u'_2 \dots u'_{|T|}$
- $(w, w') \in \mathscr{T}$
- $|(\mathsf{ustat}(w), \mathsf{ustat}(w')) - (\lambda, \lambda')|_1 \leq 2(maxout + 1)\varepsilon$

Sources of non ε-equivalence

1. statistical domains ε-far

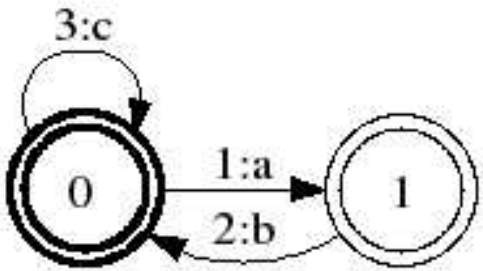


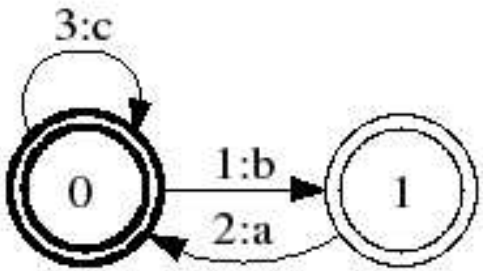


2. statistical images ε-far
3. generated length asymptotically different

Sources of non ε-equivalence

1. statistical domains ε-far
2. statistical images ε-far

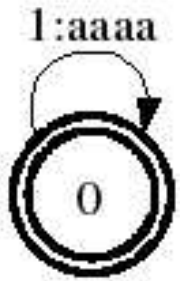




3. generated length asymptotically different

Sources of non ε-equivalence

1. statistical domains ε-far
2. statistical images ε-far
3. generated length asymptotically different



Sources of non ε-equivalence

1. statistical domains ε-far
2. statistical images ε-far
3. generated length asymptotically different

Solutions :

1. the domain is regular: similar embedding of a regular language: PTIME tester.
2. use the embedding
3. generating functions (rational functions)

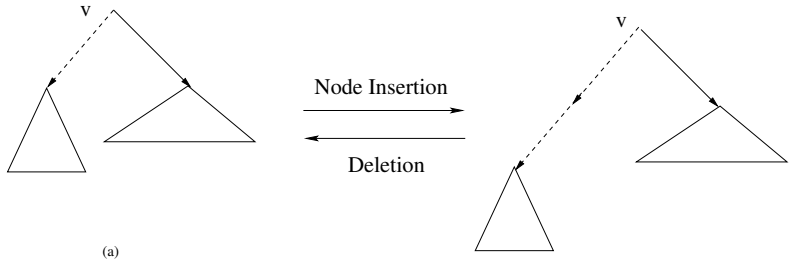
Outline: Extention to trees

8 Distances and Statistics
- Tree Edit Distance with Moves

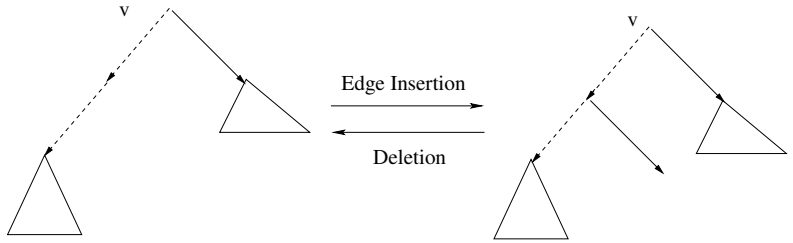
9 Tree Transducers
- Definition
- Example
- the next step

Tree Edit Distance with Moves : Elementary Operations

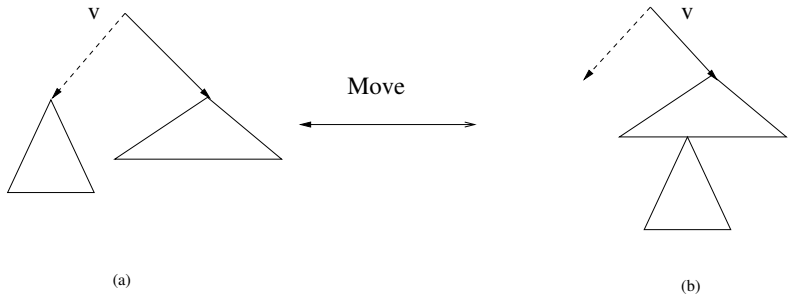
Node operation:



Edge operation:



Move:



Relaleling

Let $H_{\Sigma'}[Q]$ be the set of finite sequences of finite trees where one leaf of each tree is a distinguished element labeled by a sequence of states in Q, which is possibly empty.

Tree Transducers

A tree transducter $\mathscr{T}$ is a tuple $(\Sigma, \Sigma', Q, I, \delta)$ where

- Σ in the input alphabet, Σ' the output alphabet
- Q a set of states, $I \in Q$ an initial state
- $\delta : \Sigma \times Q \to H_{\Sigma'}[Q]$,

An xlst program

```
<xsl:output method="xml" indent="yes"/>

<xsl:template match="/">
    <root>  <s> <summary/>
                <xsl:apply-templates mode="q0"/>
            </s>
            <c> <xsl:apply-templates mode="q1"/>
            </c>
    </root>
    </xsl:template>

  </xsl:template match="a" mode="q0">
    <a/>
  </xsl:template>

  </xsl:template match="b" mode="q0">
    <b/>
  </xsl:template>

  </xsl:template match="a" mode="q1">
    <a>
       <xsl:apply-templates mode="q1"/>
    </a>
  </xsl:template>

  </xsl:template match="b" mode="q1">
    <b>
       <xsl:apply-templates mode="q1"/>
    </b>
  </xsl:template>
```

1. define tree statistics as matrices
2. Matrices distances & Edit distance
3. applications for xslt programs

thank you

$\mathcal{Q}$uestions ?

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